Find a homogeneous linear differential equation with constant coefficients whose general solution is $y = Ae^{2t} + Be^{2t}\cos t + Ce^{2t}\sin t$. Write your final answer using $y', y'', \ldots$ notation, not D notation. **SCORE: _____ / 4 PTS**

$r = 2, \underline{2 \pm i}$

$(r-2)((r-2)^2+1) = 0$

$\underline{(r-2)}\underline{(r^2-4r+5)} = 0$

$r^3 - 6r^2 + 13r - 10 = 0$

$\underline{y''' - 6y'' + 13y' - 10y = 0}$

EACH UNDERLINED ITEM WORTH 1 POINT UNLESS OTHERWISE INDICATED

Find the general solution of $4x^2y'' + 8xy' + 5y = 0$ on the interval $(0, \infty)$. **SCORE: _____ / 4 PTS**

$\underline{4r^2 + 4r + 5 = 0}$

$r = \dfrac{-4 \pm \sqrt{-64}}{8}$

$= \dfrac{-4 \pm 8i}{8}$

$= \underline{-\tfrac{1}{2} \pm i}$

$y = \underline{Ax^{-\frac{1}{2}}\cos \ln x} + \underline{Bx^{-\frac{1}{2}}\sin \ln x}$

Find the general solution of $2y''' + 7y'' + 8y' + 3y = 0$. **SCORE: _____ / 6 PTS**

$2r^3 + 7r^2 + 8r + 3 = 0$

$$\begin{array}{r|rrrr} -1 & 2 & 7 & 8 & 3 \\ & & -2 & -5 & -3 \\ \hline & 2 & 5 & 3 & \boxed{0} \end{array}$$

$\underline{(r+1)(2r^2+5r+3) = 0}$ $1\tfrac{1}{2}$

$\underline{(r+1)(2r+3)(r+1) = 0}$ $1\tfrac{1}{2}$

$r = -1, -1, -\tfrac{3}{2}$

$y = \underline{Ae^{-t}} + \underline{Bte^{-t}} + \underline{Ce^{-\frac{3}{2}t}}$

OK IF YOU USED x OR t

$y_p = 1 - 4x$ is a particular solution of $y'' - 2y' - 8y = 32x$. **SCORE: _____ / 9 PTS**

[a] Using linearity, find a particular solution of $y'' - 2y' - 8y = 16x$.

$16x = \frac{1}{2}(32x) \longrightarrow y = \frac{1}{2}(1-4x) = \frac{1}{2} - 2x$

[b] By inspection, find a particular solution of $y'' - 2y' - 8y = 1$.

$-8y = 1 \longrightarrow y = -\frac{1}{8}$

[c] Using superposition and linearity, find a particular solution of $y'' - 2y' - 8y = 16x + 12$.

$y = \frac{1}{2} - 2x + 12(-\frac{1}{8}) = -1 - 2x$

[d] Find the general solution of $y'' - 2y' - 8y = 16x + 12$.

$r^2 - 2r - 8 = 0$
$(r-4)(r+2) = 0$
$r = 4, -2$

$y = -1 - 2x + Ae^{4x} + Be^{-2x}$

FOR SOLUTION TO HOMOGENEOUS EQUATION

FOR ADDING SOLUTIONS TOGETHER

[e] Solve the initial value problem $y'' - 2y' - 8y = 16x + 12$, $y(0) = 1$, $y'(0) = -12$.

$y(0) = -1 + A + B = 1 \longrightarrow A + B = 2$ ($\frac{1}{2}$)

$y' = -2 + 4Ae^{4x} - 2Be^{-2x}$

$y'(0) = -2 + 4A - 2B = -12 \rightarrow 4A - 2B = -10$ ($\frac{1}{2}$)

$2A - B = -5$

$\longrightarrow 3A = -3 \longrightarrow A = -1$, $B = 3$

$y = -1 - 2x - e^{4x} + 3e^{-2x}$

$y_1 = x$ is a solution of $x^2 y'' - x(x+2)y' + (x+2)y = 0$. Find a second linearly independent solution. **SCORE: _____ / 7 PTS**

$y_2 = vx$
$y_2' = v'x + v$
$y_2'' = v''x + 2v'$

$x^2 y_2'' - x(x+2)y_2' + (x+2)y_2$
$= x^2(v''x + 2v') - x(x+2)(v'x + v) + (x+2)vx$
$= v''(x^3) + v'(2x^2 - x^2(x+2)) + v(-x(x+2) + x(x+2))$
$= x^3 v'' - x^3 v' = 0$

LET $u = v'$

$\cancel{x^3} u' - \cancel{x^3} u = 0$

$\frac{du}{dx} = u$

$\int \frac{du}{u} = \int dx$

$\ln|u| = x$

$u = e^x$

$v' = e^x$

$v = e^x$

$y_2 = xe^x$